Indian Statistical Institute
B. Math. Hons. II Year
Semestral Examination 2002-2003 (Backpaper)
Algebra IV
Duration: 3 Hours

Marks: 100   Instructor: B. Bagchi

Note: Each question carries 20 marks. Answer all the questions.

1. For any positive integer $n, w(n)$ denotes the number of distinct prime factors of n. If $w(n) \leq 2$ then show that any group of order n is solvable. Show by an example that this is false for $w(n) = 3$.

2. Prove that the number of inequivalent irreducible representations of a finite group equals the number of its conjugacy classes.

3. State and prove the branching theorem.

4. If a finite group G has irreducible representations π_1, π_2 on the vector spaces V_1 and V_2 then determine all the invariant subspaces of $V_1 \oplus V_2$.

5. Let λ, μ be two partitions of n, with corresponding specific characters $[\lambda], [\mu]$ for S_n. Show that the following statements are equivalent:-
(i) $[\mu](x) = sgn\ (x) \cdot [\lambda](x)$ for all $x \in S_n$, and
(ii) λ and μ are conjugate partitions.